TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

———

No. 516

———

PROPELLER VIBRATIONS AND THE EFFECT OF

THE CENTRIFUGAL FORCE

By T. Theodorsen
Langley Memorial Aeronautical Laboratory

———

Washington
February 1935

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 516

PROPELLER VIBRATIONS AND THE EFFECT OF

THE CENTRIFUGAL FORCE

By T. Theodorsen

SUMMARY

A method has been devised for determining the fre-
quencies of the various modes of a stationary propeller
and the associated crankshaft. A method has also been
devised to obtain the effect of the centrifugal force on
a revolving propeller by the use of a flexible model.

INTRODUCTION

A noticeable increase in the number of propeller
failures has recently focused the attention rather strong-
ly on this problem. The status of the knowledge on this
question will be briefly indicated in this paper. Some
of the recent work by the National Advisory Committee for
Aeronautics is presented and an attempt made to give cer-
tain tentative recommendations with respect to future
practice, insofar as this is definitely possible. Propel-
ler failures may apparently be subdivided into two class-
es: shank failures and tip failures. The various modes
of vibration that a propeller is capable of performing are
first described and convenient methods of determining them
explained.

In a first approximation, the propeller is a tapered
beam symmetrical with respect to the hub axis. A beam
supported in the middle is capable of performing two types
of vibrations, which shall be referred to as "symmetrical"
and "nonsymmetrical". The three lowest modes of each type
are indicated schematically in figure 1.

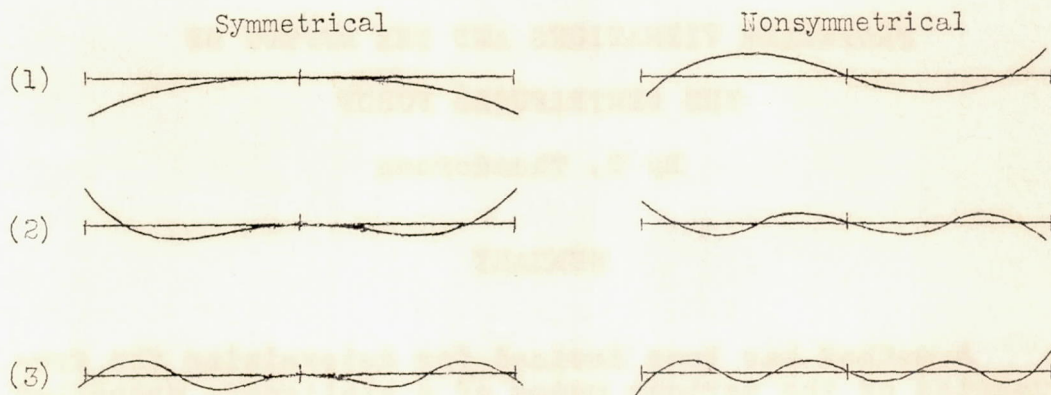   Symmetrical   Nonsymmetrical

Figure 1.

In regard to the symmetrical type of vibrations it should
be noticed (fig. 1) that the point of symmetry is fixed or
prevented from performing a perpendicular motion. This
condition is strictly in accordance with the situation of
a propeller mounted on the engine. The fore-and-aft mo-
tion of the propeller hub is of the order of some thous-
andths of an inch, while the motion of the tip in any case
of concern is several hundred times larger. With the cen-
ter fixed or free, the difference in the frequency of vi-
bration is quite considerable in the case of a rectangu-
lar beam, for which the frequencies are known. (See ref-
erence 1.)

     Figure 2 gives a comparison of the two cases, includ-
ing the frequency formulas. The free beam is therefore
seen to execute frequencies that are higher than those of
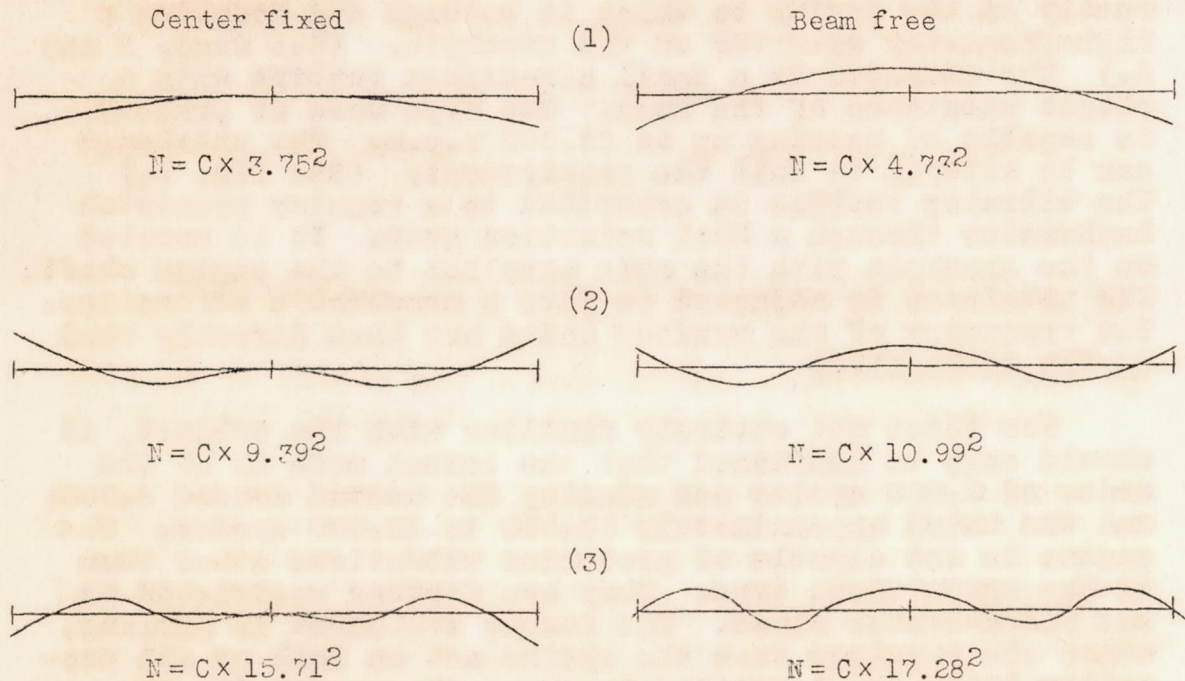the beam fixed at the center by the following factors:

               First mode: 1.61

               Second mode: 1.38

               Third mode: 1.21

The results are illustrative, but of course almost mislead-
ing as to magnitude, as the effect is much less pronounced
on a tapered beam or a propeller, in particular with re-
spect to the third mode, which is ordinarily of most con-
cern. The following discussion will be restricted to the

type with the center fixed, as being the only one of in-
terest.

Center fixed   (1)   Beam free

$N = C \times 3.75^2$ $N = C \times 4.73^2$

(2)

$N = C \times 9.39^2$ $N = C \times 10.99^2$

(3)

$N = C \times 15.71^2$ $N = C \times 17.28^2$

$$C = \frac{K}{2\pi L^2}\sqrt{\frac{E}{\rho}}$$ L = length of beam.
K = the radius of gyration

Figure 2

    In order to create this type experimentally, it is
very convenient to mount the propeller on a shaft and
then subject the shaft to an alternating torsional twist-
ing moment. To obtain the twisting moment, the National
Bureau of Standards has employed an electric motor mount-
ed on the shaft and supplied with an alternating current
of the desired frequency (reference 2). The N.A.C.A. has
adopted a slightly different scheme, convenient because
of its simplicity and the possibility of obtaining as
many of the higher modes as are desired, usually as high
as the fifth order.

MEASUREMENT OF PROPELLER FREQUENCIES

This method consists in mounting the propeller directly on the engine to which it belongs and mounting a high-frequency vibrator on the crankpin. (See figs. 3 and 4.) The vibrator is a small air-driven turbine with a slight unbalance of the mass. The type used at present is capable of running up to 30,000 r.p.m. The unbalance can be altered to suit the requirement. (See fig. 5.) The vibrator turbine is connected to a regular precision tachometer through a 20:1 reduction gear. It is mounted on the crankpin with the axis parallel to the engine shaft. The unbalance is adjusted to give a measurable deflection. The frequency of the various modes are then directly read on the tachometer.

For those not entirely familiar with the subject, it should only be mentioned that the lowest mode is of the order of 2,000 cycles per minute; the second around 6,000; and the third approximately 10,000 to 12,000 cycles. The engine is not capable of producing vibrations other than of the symmetrical type. They are further restricted to the fixed-center class. The former statement is obvious, since the impulses from the engine act on both or all propeller blades in identical manner. The reason for the second statement has already been given.

The vibration frequencies have been obtained at the Langley Memorial Aeronautical Laboratory on a number of conventional propellers; but, since the experiment can very easily be reproduced, these are of little general interest. As is already known, the angle of pitch has a noticeable influence on the frequency. The effect is not very large, however, through the ordinary range of pitch angles.

Regarding the location of the various nodes, these may readily be obtained either by the dust method commonly used or by a vibration-amplitude recorder of the type described in reference 3, which method is used by the Laboratory. It should be emphasized, however, that an exact determination of these nodal points serves no particular purpose since, first, they are considerably altered by the effect of the centrifugal force and second, because a propeller is not expected to be operated in a state of critical vibration, as shall be pointed out more clearly. Only as a matter of explaining or analyzing definite cases or

propeller failures are these nodal points of interest, as they help to establish, to a certain degree, the exact location of expected maximum strains. In fact, the interest in the strain due to vibration is only for the purpose of explaining why certain failures occurred; thus learning how to avoid certain dangerous conditions.

The following paragraphs will be devoted to a discussion of how to alter the shape of the propeller so as to obtain more or less specified vibration frequencies; or, in other words, to point out how any one of the various modes may be made to appear at higher or lower frequencies.

The frequency of a certain mode of vibration is given by the formula

$$N = \frac{D}{2 \pi L^2} \sqrt{\frac{E}{\delta}} \, f(s) \quad \text{(See reference 1, page 361)},$$

where L is the length of the propeller blade and D is a quantity representative of the thickness, say the diameter at the hub; $f(s)$ is a function of the relative shape of the propeller and depends also on the mode. The number $f(s)$ is the same for all <u>similar</u> blades; that is, similarity in cross section for similar locations along the length L. Starting with a certain original propeller, all dimensions along the length axis may be increased in a certain proportion, or all dimensions in the plane perpendicular to the length axis be increased in a certain ratio, or both. The function $f(s)$ will not be altered by this process. Note that this kind of similarity differs from the ordinarily used definition in the fact that two factors of proportionality are involved, one giving the ratios of cross sections and the other the ratio of lengths. The resulting "similar" bodies are all perfectly orthodox propellers, and furthermore it can be shown that all types of propellers are rather closely related so that the function $f(s)$ is not very appreciably altered.

The first important rule is: The frequency of the various modes of vibration varies directly with the thickness and inversely with the square of the length.

If an increase in the frequency of any particular mode is desired (a problem frequently encountered), and this result is not wanted at the expense of an increase in weight of the propeller or an alteration in its principal dimensions, the procedure of reasoning is as follows:

The frequency of a certain mode is dependent
on the ratio of the energy stored in the deflected
structure in the particular mode to a certain ef-
fective mass of the structure. Near the nodal
points, stored energy is small since the curvature
is small. Near the loops, the stored energy is con-
siderable and is proportional to the square of the
curvature (or deflection), and to the quantity EJ.
The frequency may be increased by increasing the
stored, or potential, energy in the region near the
greatest curvature. A second rule can therefore be
expressed very simply:

To increase the frequency of vibration of a
certain mode increase the stiffness at points near
greatest curvature (loops) by removing mass from
points near small curvature.

In order to illustrate the method, refer to figure
6 showing the third mode of vibration (symmetrical, cen-
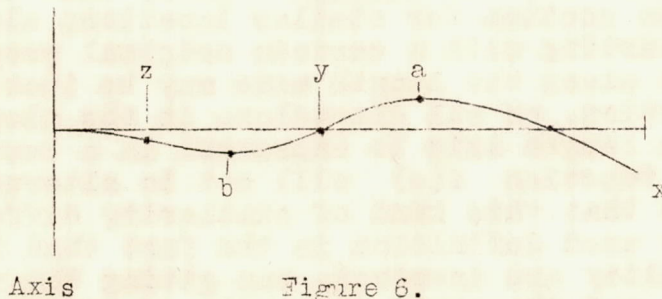ter fixed).

Axis   Figure 6.

Note that points of greatest curvature are a, b, and some
point c near the hub; the points of least curvature are
the tip x, some point near the middle node at y, and
the inner point z. Because of the small deflections near
the hub, the central portion has a very small effect on
the frequency. The procedure in raising the frequency of
the third harmonic consists primarily in removing mass
from the tip to the region near a in the middle of the
outer loop, and secondarily by removing mass from the
point y near the middle node to the point b in the
middle of the inner loop. It should be noted in addition

that the thickness-chord ratio at a and b should be as
large as permissible, since the object simply is to in-
crease the moment of inertia around a line in direction of
the chord.

The nodes measured on a 9-foot propeller at the Lab-
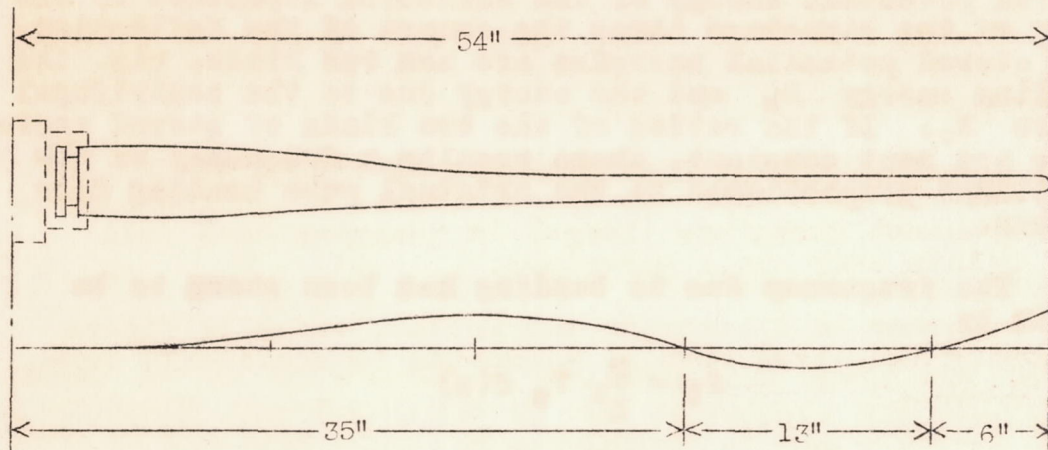oratory for the third mode are given as an illustration
in figure 7.

Figure 7.

It is evident from the figure that, in order to
raise the frequency of the third-order vibration, the
particular propeller shown should be thickened at the
stations: approximately 41 inches, 25 inches, and hub.
In order to save weight it should be thinned out at the
stations: tip (54), 35, and approximately 15.

The identical or reverse procedure may be applied to
lower or raise any one of the various frequencies.

THE EFFECT OF CENTRIFUGAL FORCE ON PROPELLER VIBRATIONS

Elementary considerations show that the centrifugal
force acting on a revolving propeller increases the vibra-
tion frequencies of the various harmonics. The question
then is how far the frequencies obtained on a nonrevolving
propeller may be used to consider actual conditions. It
shall be shown that the effect is considerable. It is
subject to a rather straightforward computation, which,
however, in the case of a true propeller shape becomes

very laborious. The method is given by Hohenemser (reference 4), who carries through the calculation for the three lowest modes on a simplified (triangle shaped) length section. The author has devised a very convenient method for determining the effect. It is based on the following reasoning.

The frequency in any mode depends on the ratio of stored potential energy of the deflected structure to the mass of the structure times the square of the deflection. The stored potential energies are now two kinds, viz, the bending energy E_b and the energy due to the centrifugal force E_c. If the ratios of the two kinds of stored energies are kept constant, there results a frequency of vibrational proportional to the original pure bending frequency.

The frequency due to bending has been shown to be given by

$$f_b = \frac{D}{L^2} V_s \, f(s)$$

where $V_s = \sqrt{\dfrac{E}{\rho}}$ (sound velocity in the material). The frequency due to centrifugal force alone is given by

$$f_c = w \, f_1(s) \quad \text{(see reference 1, page 367)},$$

where w is the frequency of revolution and $f(s)$, like $f_1(s)$, depends only on the shape and is definite number for the defined similar propellers.

The resulting frequency in any particular mode $y = f(x)$ is given by

$$f^2 = f_b{}^2 + f_c{}^2$$

By the simple expedient of keeping the ratio of the expressions f_b to f_c constant, a mathematically correct reproduction of the relative frequencies is obtained. This ratio is for similar propellers, except for a constant

$$R = \frac{D}{L^2} \frac{V_s}{w} = \frac{D}{L} \frac{V_s}{V_T}$$

where V_T is the tip velocity.

The rule is obtained:

The relative increase in frequency due to centrifugal force in _similar_ propellers is the same for the same value of the quantity $R = \frac{D}{L} \frac{V_S}{V_T}$.

This fact is very useful. It is obviously very difficult to obtain experimental results on a full-size propeller. Identical increases due to the centrifugal force are obtained by reducing the cross section, however, say 10 times, and running the resulting thin propeller at one-tenth speed. For a full-scale propeller showing a frequency of 2,000 cycles per minute at rest and, say 3,000 cycles per minute at 2,000 r.p.m., there is obtained on the thin reproduction 200 cycles per minute at rest and 300 cycles per minute at 200 r.p.m.

The experiment is very easy to perform on this thin copy. The horsepower requirement is down to one thousandth of the former value. It was found desirable to enclose the entire propeller in a protecting tube to prevent the air damping from influencing the results.

The experimental installation is shown in figure 8 with the tube off and in figure 9 with the tube on. The propeller is mounted horizontally to avoid any effects of gravity. A vibrator is mounted near the hub to impose vibrations at any desired frequency in a vertical direction. Critical frequencies make themselves evident by the propeller tip hitting the metal tube, which acts as a bell. The propeller is run at a certain revolution speed and the vibrator is gradually speeded up until the sound from the tube is heard. The maximum is carefully adjusted and noted.

The results of the first experiment of this type are given in figure 10. The abscissa is here the revolution speed of the propeller and the ordinate is the vibration frequency. (Both values have been multiplied by 10.) Although the results may be considered somewhat preliminary they are in remarkably good agreement with those by Liebers (references 5 and 6) for the fundamental and with the values calculated by Hohenemser (reference 4) for the second and third harmonics. Hohenemser gives for the value c in the usual formula $f = \sqrt{f_0^2 + c\ N^2}$ the values $2.5 < c < 3.9$ for the second and $c < 12.2$ for the third,

both calculated for the simplified case of the triangular
length section perpendicular to the blade and constant
width. The Laboratory experiment gives, for comparison,
the values indicated in parentheses in the figure and re-
produced in the following:

<div align="center">Values of   c</div>

	N.A.C.A. measured experimentally	Hohenemser calculated for triangular length section
1st mode	1.7	--
2d mode	2.8 - 3.5	2.5 - 3.9
3d mode	12 - 12.3	< 12.2
4th mode	21 - 24	--

On the basis of the experience gained, it is the in-
tention to refine the method to obtain greater accuracy.
In particular the responses were not quite as definite as
they could have been, owing to a damping effect of the
rather heavy auxiliary supporting structure. A new design
is under construction. It is planned to study the effect
of particular shapes in more detail.

<div align="center">THE EFFECT OF UNDESIRABLE PROPELLER

CRANKSHAFT COMBINATIONS</div>

Figures 11 and 12 show typical cases, each with two
different propellers. The curved lines are the successive
propeller modes, reconstructed by means of the experimen-
tal values obtained for c, and the horizontal line gives
the measured crankshaft critical.

The latter is obtained in the same type of experiment
as already described for the determination of the station-
ary propeller frequencies. Equivalent weights are used in
the crankpins to replace the piston system. The auxiliary
drives have also been removed. This procedure results in
a very sharp and well-defined shaft critical. This value

was found to be 9,800 cycles per minute for the P. & W.
Wasp, corresponding to 2,180 r.p.m. for the 9-cylinder
engine, and at 12,600 r.p.m. or 1,800 r.p.m. for the 14-
cylinder 2-row R-1830 engine. The former result is in
very good agreement with results obtained at Wright Field
with the Prescott Indicator (reference 7, page 3). The
second result appears to be 100 - 200 cycles per minute
higher than similar results obtained by the Prescott In-
dicator. The present experiment was performed mainly to
demonstrate the method, and no attempt will be made to
explain minor discrepancies.

The conclusions to be drawn from the propeller-engine
characteristics shown in figures 11 and 12 are that the
former represents a desirable combination in the range
1,500 - 2,180 r.p.m. for one propeller and 1,300 - 2,180
for the second. The range could be extended downward,
however, by employing a more flexible propeller than ei-
ther of the two types shown. It is seen that the second
mode goes into resonances with the explosion impulses of
the engine at 1,500 and 1,300 for the two propellers, re-
spectively, and that the third mode even for the second
propeller (dotted) is beyond the range entirely. Conclu-
sions from the second diagram (fig. 12) are that this is
a very undesirable combination. At 1,800 r.p.m., not on-
ly the crankshaft critical but also the third mode of both
propellers are in resonance with the explosion frequency
of the 14-cylinder engine. Remedy: - Employ a propeller
with the third mode at about 12,500 cycles per minute,
and lower the shaft critical speed (if necessary by means
of spring hub) to about 8,500 cycles per minute. This
procedure will leave the range 1,250 to 2,000 r.p.m.
clear. Diagrams of this type are considered indispensable
in connection with aircraft-engine propeller installations.

PROPELLER FAILURES

When the explosion frequency is in resonance with the
shaft critical speed, there results a variable torque of
considerable magnitude. From considerations of the moments
of inertia and the known frequencies it is found that the
twist of the crankshaft due to the full-load torque of the
engine is of the order of $1/4°$ for the radial types in-
vestigated. Let it be assumed that the variable torque
amplitude is about one quarter of this average value, or
about $1/16°$. Now deflections have been observed in oper-

ation of 1° and even as much as 2°, which means that the
variable torque due to resonance has been increased by a
factor of 16 to 32. This torque enters the propeller
shank as a bending moment and, if the values given are
fairly representative, the shank is subjected to an alter-
nating bending moment of from 16 to 32 times the static
value, or from 4 to 8 times the normal average bending
moment resulting from the air load. It is obvious that
the condition may be dangerous, insofar as the endurance
limit may be exceeded. If the shaft is not running at its
critical speed but in resonance with one of the propeller
modes the strain is at normal value near the hub; whereas
a tip breakage may occur.

The condition of double resonance, as was indicated
to occur in the case reproduced in figure 12, is still
more dangerous as far as a tip failure is concerned, al-
though it will depend on the relative circumstances wheth-
er a shank or a tip failure will result.

Langley Memorial Aeronautical Laboratory,
     National Advisory Committee for Aeronautics,
          Langley Field, Va., January 18, 1935.

REFERENCES

1. Grammel, R.: Mechanik der Elastischen Korper. Hand-
      buch der Physik, vol. VI, Julius Springer, Berlin,
      1928.

2. Dryden, Hugh L., and Tuckerman, L. B.: A Propeller-
      Vibration Indicator. Bur. Standards Jour. Research,
      May 1934, pp. 537-42.

3. Theodorsen, T., and Gelalles, A. G.: Vibration Response
      of Airplane Structures. T.R. No. 491, N.A.C.A., 1934.

4. Hohenemser, K.: Beitrag zur Dynamik des elastischen
      Stabes mit Anwendung auf den Propeller. Z.F.M.,
      Jan. 28, 1932, vol. 23, no. 22, pp. 37-43.

5. Liebers, F.: Contribution to the Theory of Propeller
      Vibrations. T.M. No. 568, N.A.C.A., 1930.

6. Liebers, F.: Resonance Vibrations of Aircraft Propel-
      lers. T.M. No. 657, N.A.C.A., 1932.

7. Prescott, F. L.: Vibration Characteristics of Aircraft
      Engine Crankshafts. A.C.I.C., vol. VII, no. 664,
      Air Corps, Matériel Division, Wright Field, 1932.
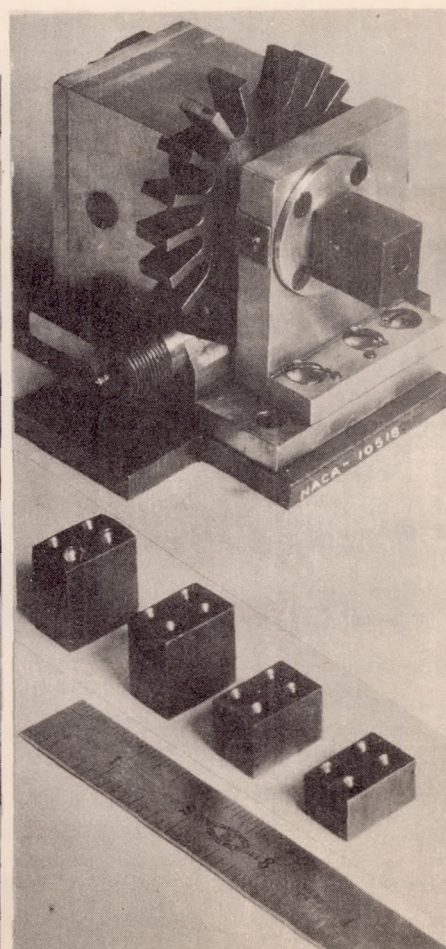
14.
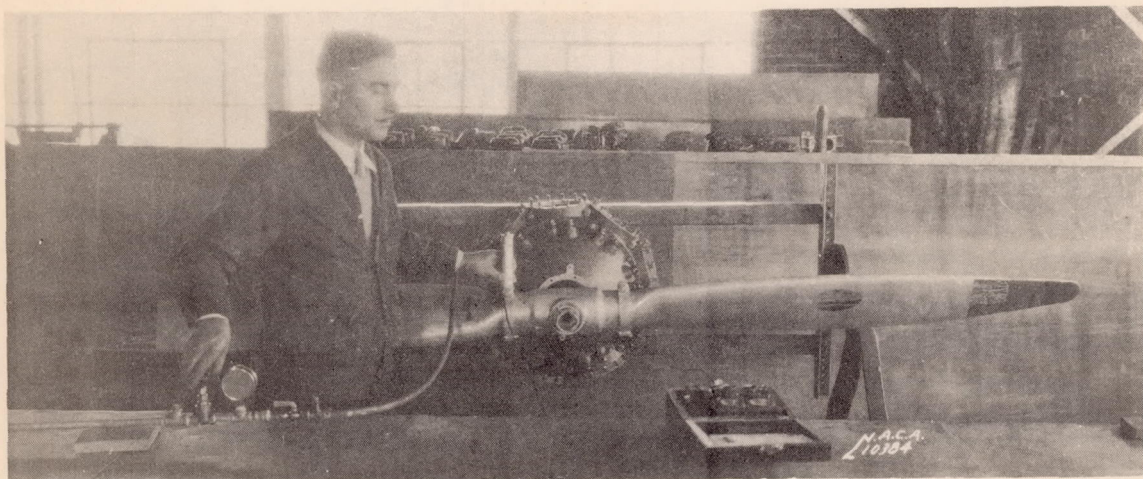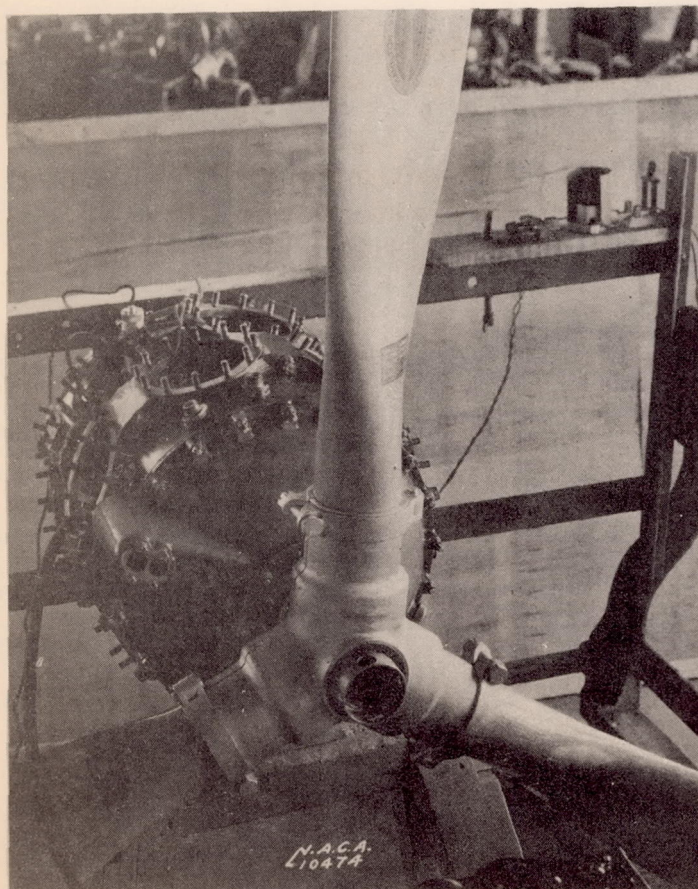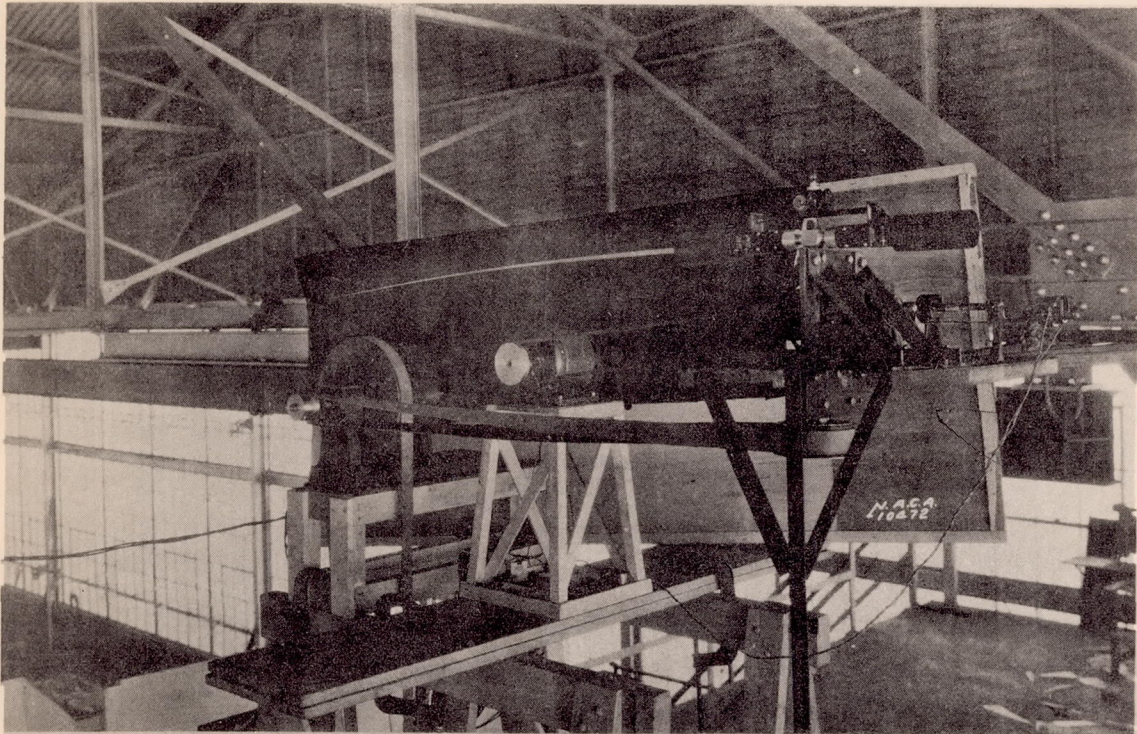

Figure 3.

Figure 4.

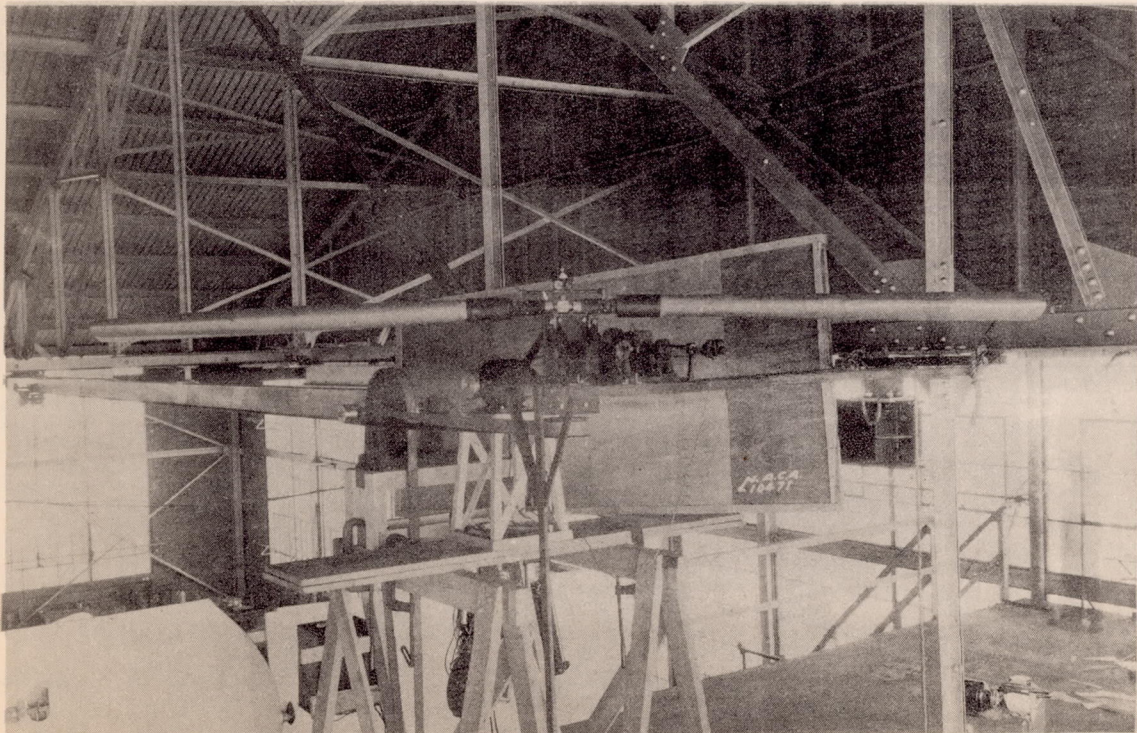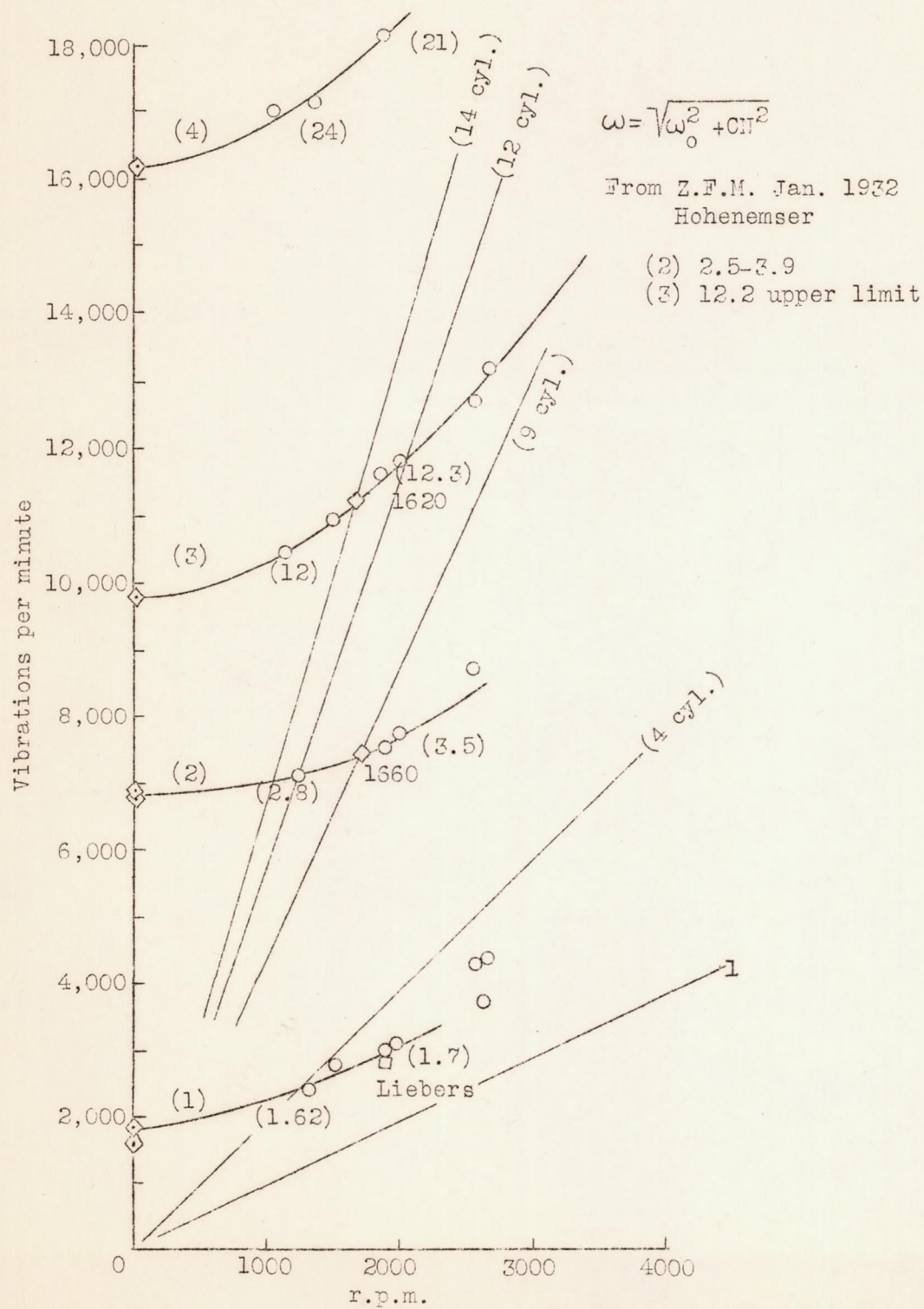Figure 5.

15.

Figure 8.

Figure 9.

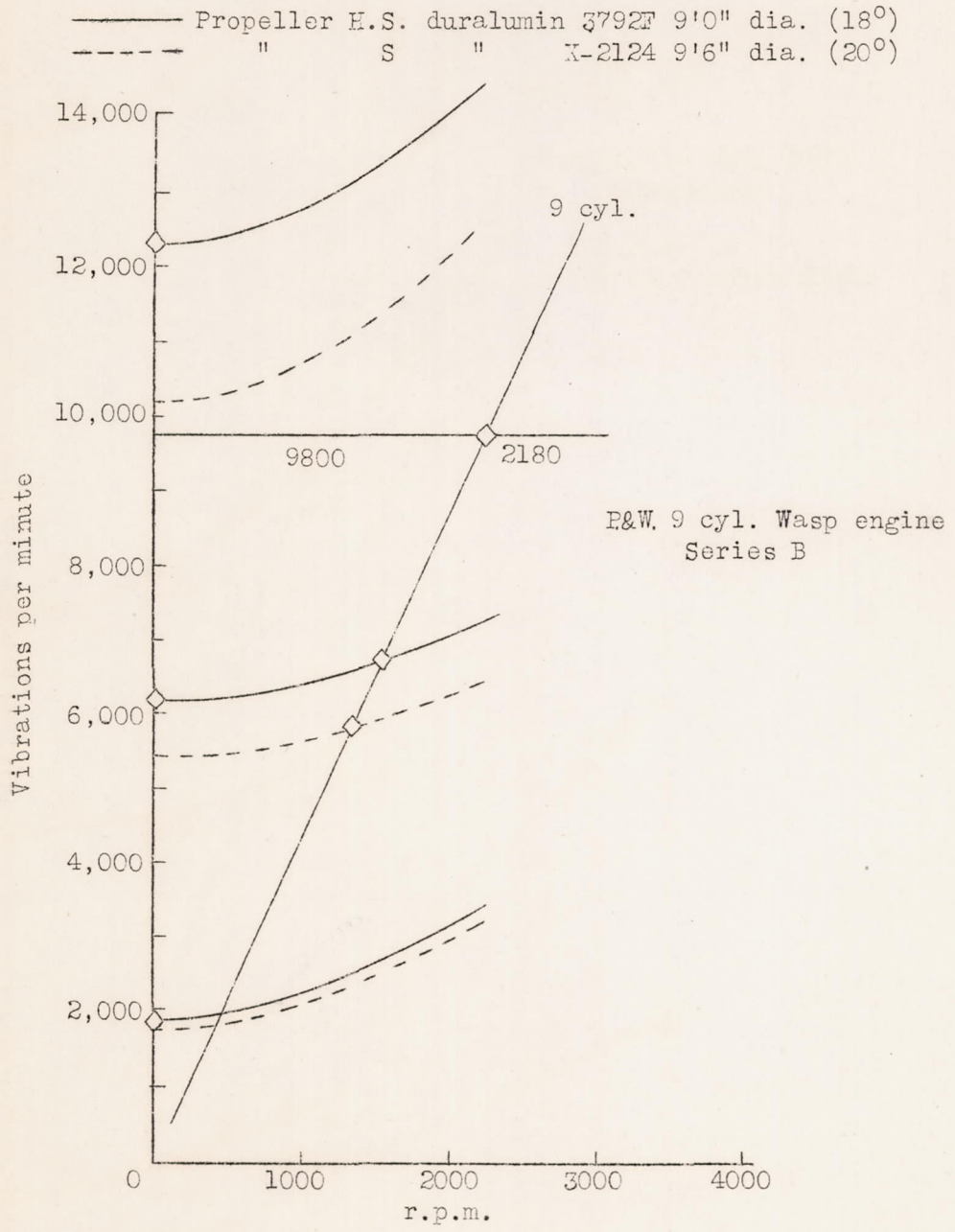Figure 10.-Effect of centrifugal force on frequencies of propeller vibrations.

17.

Figure 11.-Propeller crankshaft vibrations.

——————Propeller 3 blade H.S. duralumin, drawing 5789G 9'0" CM (14°)
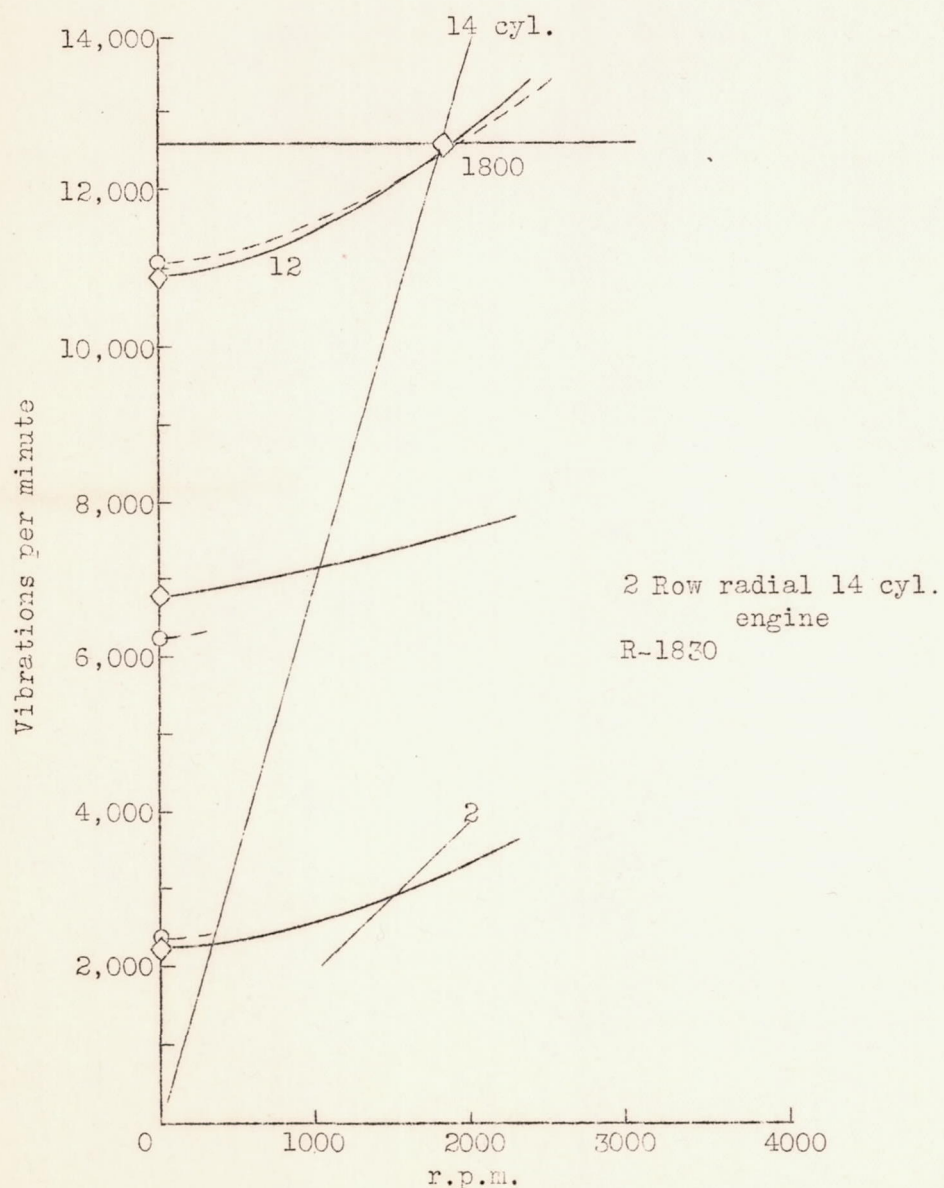– – – – –   "   "   "   Pitt steel, drawing 400 9'0"

Figure 12-Propeller crankshaft vibrations.